SPATIAL SPECTRUM ANALYSIS OF VARIOUS DIGITAL ELEVATION MODELS

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ABSTRACT

Digital elevation models (DEM) are baseline geospatial products widely used in mapping and other applications. Generally obtained from airborne sensor data, DEMs can be made available at various spatial resolutions and accuracy levels. As DEM production costs vary significantly depending on the detail level of these two parameters, plans for data acquisition must be carefully balanced so as to satisfy spacing and accuracy requirements at the lowest possible cost. While most standard mapping techniques provide methods for determining the required resolution, they are usually based on qualitative surface definitions such as open, built-up and/or urban terrain, or by land cover such as weeds, crops, scrub, wooded, etc. These categories, however, are not necessarily correlated to the complexity of the surface, which ultimately defines the spatial sampling distance for any given surface representation requirements. This paper investigates a method for surface characterization that uses the spatial spectrum. Mathematically, DEMs can be modeled as real, single-valued functions of two variables. Using a 2D Fourier transform, surfaces can be represented in the spatial frequency domain, where each frequency can be interpreted as a different sampling rate of the measurements (i.e., spacing). In a simple interpretation: low-frequency components characterize slow changes of the surface, such as average slope, while high-frequency components describe the microstructure, or local, details. Evaluating representative DEMs, the most typical spatial spectra can be determined which, subsequently, can help practitioners define the optimal sampling distances and/or error requirements to determine DEM resolution and data acquisition for major surface categories.

Key words: Spatial spectrum, spatial sampling, 2D Fourier transform, DEM

INTRODUCTION

Since DEMs (Digital Elevation Model) became a baseline mapping/geospatial product, they have been broadly used in almost all mapping and engineering (as well as many other) applications (Muane, 2007). For example, they are directly used in flood-plain mapping or for line-of-sight analysis in telecommunications. Indirectly they are used in orthophoto production or 3D city modeling. The real proliferation of DEM-based analysis started with the introduction of powerful computers and digital photogrammetric systems that could provide an affordable platform for mass surface-point generation from scanned airborne imagery as well as satellite radar acquisition able to process point clouds acquired by LiDAR imaging systems. Most recently, the introduction of SGM (Semi-Global Matching), which is widely used to process UAS (Unmanned Aerial System) imagery, has reinvigorated stereo-image-based production of DEMs (Hirschmuller, 2005).

There are numerous terms used for describing surface models and data including DSM (digital surface model), DTED (digital terrain elevation data), DTM (digital terrain model), DEM (digital elevation model), etc. Some of these overlap in definition, while others are unique. In the following, only DEM is used as a general term. Note that the term “point cloud” (PC) has been widely used in conjunction with DEMs, but shouldn’t be confused with DEMs as PCs represent a more general spatial model/data structure; DEMs are single-valued 2D functions, while PCs have no limitation on the 3D point distribution.

Government organizations, data vendors, and business clients make various specifications on these products in terms of accuracy, reliability, etc., that are defined as regularizations, quasi-standards, guidelines, etc. The regulations most often used in the US are from federal and/or non-profit agencies such as USGS, FEMA, NGA, FGDC, ASPRS, etc. All of these standards/guidelines are focused mainly on aspects of the DEM including accuracy level, ground control and statistical evaluation methods. Little or no attention has been paid to characteristics of the actual surface, or, in a broader sense, to what impact the complexity of the object space has on DEM characterization.

The question is how the terrain can be characterized using simple and straightforward methods. In other words, a classification of what category of terrain complexity corresponds to a specific surface that is based not just on the experts' subjective decision, but formalized in a mathematical way to optimize the measurement planning and the QA/QC processes. The DEM spatial spectrum provides a potential tool for analyzing surface complexity. In theory, DEMs can be modeled as real, single-valued functions of two variables. Using 2D Fourier transform, the surfaces can be represented in the spatial frequency domain, where each frequency can be interpreted as a different sampling rate of the measurements (i.e., spacing). In a simple interpretation: low-frequency components characterize the slow changes in the surface, such as average slope, while high-frequency components describe the microstructure, or local, details. Evaluating representative DEMs, the typical spatial spectra are determined which, subsequently, can help practitioners define optimal sampling distance and/or error requirements for DEM resolution and data acquisition for major surface categories.

This paper is a part of an ongoing research project. The objective of this study is to investigate the complexity of the surface/object space condition in terms of spatial sampling of the surface and to categorize surfaces to analyze the impact of these categories on the QA/QC processes of DEM data. First, in earlier work, the idea behind the downsampling and the filtering of frequency domains was introduced (Toth, 2011). Then this method was applied to LiDAR point-cloud-derived DEMs (Toth and Grejner-Brzezinska, 2012). In this study, three land surface types are analyzed and reconstruction error at different downsampling rates is calculated. The results show a relationship between the sampling rate and achievable accuracy level. The applied method makes no distinction with respect to the origin of the DEM; it can come from stereo or multiple ray imagery or LiDAR point cloud data as well as radar scans. In this study, elevation data derived from the SRTM (Shuttle Radar Topography Mission) was used.

BACKGROUND AND THEORY

From the DEM point of view, the reconstruction ability of any data acquisition technology is determined by the sampling frequency and the accuracy of the measurements. In the case of sampling frequency, for example, with increased sampling frequency, the sampling distance will decrease, resulting in a finer representation of the surface; with lower sampling frequency, the sampling distance will grow providing a coarser approximation the surface. Theoretically, the ideal sampling frequency for a given surface complexity is the Nyquist rate. On the other hand, accuracy (vertical measurement error) also has an impact on the DEM: with large vertical errors, the details in the reconstructed DEM will decrease or even disappear. In practice, the aim is to determine the largest sampling rate where the reconstruction error is equal or comparable to the accuracy of the vertical measurement.

Sampling

Let the land surface be given as a 2D function:

$$z = s(x, y), \tag{1}$$

where $s(x, y)$ is the continuous "real" surface. Discretizing the continuous function at x_i, y_i points, one realization of the sampling can be:

$$z_{i,j} = s(x_i, y_j) \tag{2}$$

The arrangement of the sample points can be regular or irregular. Irregular sample points can be obtained, for instance, by conventional land surveying wherein a TIN model is created and generally used (though a grid model is frequently derived from irregular data by resampling to a regular grid). Other technologies provide nominally regular but not rectangularly arranged point sets. For example, sinusoidal or saw-tooth patterns are obtained by some type of LiDAR sensors (without terrain distortion); note that these types of data are usually resampled to a regular grid. Since almost all DEM products are distributed in regular grid format, the subsequent discussion is only concerned with that data type.

The sampling frequency is the number of the sample points in a unit distance. In the X direction, assume that the sampling distances, d_x^s, are equidistant and the number of samples, N^x, is finite. Then

$$d_x^s = x_i - x_{i+1} = const, \qquad i = 1..N^x - 1 < +\infty \tag{3}$$

and, thus, the sampling frequency can be calculated with following expression:

$$f_s^x = \frac{1}{d_x^s}, f_s^y = \frac{1}{d_y^s} \tag{4}$$

where f_s^x, f_s^y are the frequency in the X and Y directions, respectively. The aim of the data acquisition is to acquire enough information to allow for reconstruction of the original surface from the samples; the function *s(x,y)* has to be estimated from the $z_{i,j}$ samples. Note that the sampling frequency impacts the surface reconstruction results. If *s(x,y)* is a band-limited function, then there exists a highest frequency, called bandwidth, above which there are no non-zero spectral frequencies. For the 2D case, there are B^x and B^y in the X and Y directions, respectively. The double of the bandwidth is the so-called Nyquist rate (Shannon, 1949). If the sampling rate is higher than the Nyquist rate, then the surface can be reconstructed perfectly:

$$f_s^x \geq 2B^x = f_N^x \text{ and } f_s^y \geq 2B^y = f_N^y \quad . \tag{5}$$

This inequality is called the Nyquist criterion, and the surface can be reconstructed without any error as the composition of *sinus cardinalis* functions:

$$s(x,y) = \sum_{i=-\infty}^{+\infty} \sum_{j=-\infty}^{+\infty} z_{i,j} sync(\pi d_x^s (x - \frac{i}{d_x^s})) sync(\pi d_y^s (y - \frac{j}{d_y^s})) \tag{6}$$

Ideally, the bandwidth should be known in order to calculate the ideal sampling frequency. Determination of the Nyquist rate is usually accomplished by measurements and analysis of the spatial spectrum. In most geospatial data acquisition and product generation tasks, there is an expected or target sampling rate and, thus, the band limits (or approximate band limits) are known from past experience. If the band limit is unknown, then it may be estimated from a sequence of tests done with various sampling rates. Note that, with undersampling, problems may occur such as aliasing and/or spectral leakage.

Reconstruction error of downsampling

In order to characterize the impact of downsampling, i.e., the loss of signal energy, we define the reconstruction error in the spatial domain with the RMSE (root mean square error) such that:

$$RMSE(\tilde{z}) \stackrel{\text{def}}{=} \sqrt{\frac{\sum_{i,j=0}^{N^x-1,N^y-1} \left(z_{i,j} - \tilde{z}_{i,j}\right)^2}{N^x * N^y}} \tag{7}$$

where $z_{i,j}$ is the original and $\tilde{z}_{i,j}$ is the reconstructed surface.

In order to estimate the reconstruction error of downsampling, Parseval's theorem is applied to create the connection between the time and frequency domains. The expression for discrete Fourier transform is the following:

$$E_z = \sum_{i,j=0}^{N^x-1,N^y-1} \left|z_{i,j}\right|^2 = \frac{1}{N^x * N^y} \sum_{i,j=0}^{N^x-1,N^y-1} \left|Z_{i,j}\right|^2 \quad (8)$$

This equation states that energy in the time domain is equal to power in the frequency domain. The downsampling is implemented by a rectangular (rect) function defined as:

$$h_{i,j} = \begin{Bmatrix} 0, if\ n_{f_c}^x < i < N - n_{f_c}^x\ and\ n_{f_c}^y < j < N - n_{f_c}^y \\ 1, otherwise \end{Bmatrix} \quad (9)$$

that removes the frequencies inside of the region of $n_{f_c}^x$ and $N^x - n_{f_c}^x$ as well as $n_{f_c}^y$ and $N^y - n_{f_c}^y$. Here f_c is the cutting frequency and $n_{f_c^x}^x, n_{f_c^y}^y$ are the corresponding indices of the cutoff frequencies of f_c^x,f_c^y. Note that this rectangular function acts as a low-pass filter. While mathematically the filtering is not the same as downsampling, it is a good approximation.

To estimate the reconstruction error, we extend Eq. (8) with the windowed region:

$$\frac{1}{N^x * N^y} \sum_{i,j=0}^{N^x-1,N^y-1} \left|h_{i,j} Z_{i,j}\right|^2 = \frac{1}{N^x * N^y} \sum_{i,j=n_{f_c^x}^x, n_{f_c^y}^y}^{N-n_{f_c^x}^x, N-n_{f_c^y}^y} \left|Z_{i,j}\right|^2 = \sum_{i,j=0}^{N^x-1,N^y-1} \left(z_{i,j} - \tilde{z}_{i,j}\right)^2 \quad . \quad (10)$$

Note that the right side of the expression is the difference between the original and reconstructed surfaces after downsampling. Calculating the square root of Eq. (10) and dividing it by $N^x * N^y$:

$$\sqrt{\frac{1}{(N^x * N^y)^2} \sum_{i,j=n_{f_c^x}^x, n_{f_c^y}^y}^{N-n_{f_c^x}^x, N-n_{f_c^y}^y} \left|Z_{i,j}\right|^2} = \sqrt{\frac{\sum_{i,j=0}^{N^x-1,N^y-1} \left(z_{i,j} - \tilde{z}_{i,j}\right)^2}{N^x * N^y}} = RMSE(\tilde{z}_n) \quad (11)$$

gives the reconstruction error. This expression shows that the reconstruction error of downsampling can be estimated by the square root of the power spectra of the removed frequencies. Suppose that no systematic error is present in the measurements:

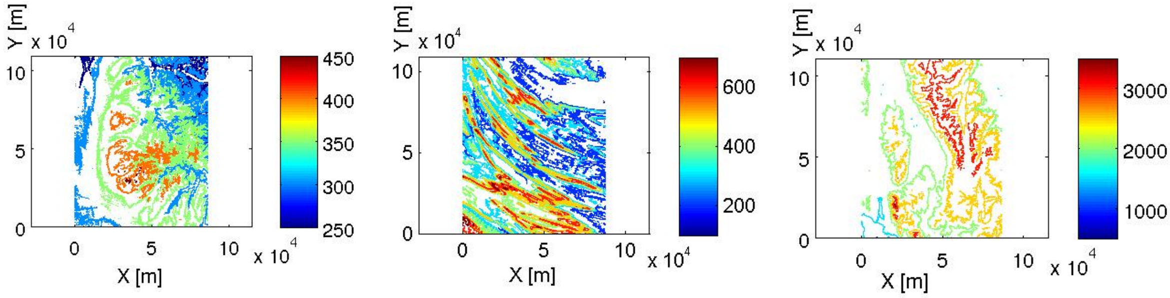
$$RMSE(\tilde{z}_n) = \sigma_m \quad (12)$$

where σ_m is the standard deviation of the measurement. This statement suggests that the measurement accuracy corresponding to a sampling rate can be calculated from the removed frequencies determined by sampling rate.

EXPERIMENTS

Three types of digital elevation models were used for analyzing the typical surfaces categories. All were derived from SRTM 1-arc dataset. The first data set is a “flat” territory from Ohio with an ~200 m difference in elevation (DEM-1). The second DEM is located on the edge of the Appalachia range in Pennsylvania with an ~600 m difference in height (DEM-2). The third DEM covers a region of the Rocky Mountains with an ~2300 m difference in range (DEM-3). All DEMs are of 3601 by 3601 raster, transformed into UTM projection, resulting in spacings of ~25 and ~30 meters in the X and Y directions, respectively. The total area of coverage is about

9725.4 km^2, or ~90,025 by ~108,030 m. The contour plots of the DEMs are shown in Figure 1. The large GSD means that the areas are strongly under-sampled, and many surface details are lost. Yet for this study, they still can be used based on our assumption that the surfaces have the proper spatial sampling. Note that these DEMs may be scaled to create a test surface with centimeter-level detail. Also note that the derived results in terms of meaning wouldn't change.



DEM-1 **DEM-2** **DEM-3**

Figure 1. Contour plots of the DEMs.

To model different sampling rates, a rectangular windowing (rect function) in the spatial frequency domain was applied to the DEMs, implementing a low-pass filter. The basic processing steps are illustrated in Figure 2. First, the DEMs are transformed into the frequency domain with 2D discrete Fourier transform. Then the spectrum is windowed with the desired frequency cut (see Figure 2a-2b). The error of the surface reconstruction can be calculated based on Eq. (11), providing an estimate of the RMSE. Next the surface is reconstructed based on the truncated spectrum (see Figure 2d). Since the higher frequency components have been removed, the surface is not as accurate and smooth as it was before applying the resampling method. The contours of the reconstructed DEM are sharper due to the use of the rect function. Note that filters such as Hamming, Hann, Parzen, etc., can be used to decrease the impact of this error. This effect can cause a small deviation between the estimated RMSE from the frequency domain and the RMSE in the spatial domain (Eq. (8)).

Applying different cutoff frequencies to the DEMs, different downsampling is realized and, subsequently, the RMSE of the downsampling can be estimated. In these tests, 35-, 40-, 45-, 50-, 80-, and 100-meter downsampling rates were used for all the DEMs. The reconstruction errors, in linear and logarithmic format, are shown in Figure 3.

As expected, DEM-3 is more sensitive to changes in the downsampling rate due to its more complex surface. Figure 3 clearly shows the error increasing as it is introduced by the larger sampling distances. Thus this technique can provide good information for data acquisition planning with specific accuracy requirements. For instance, if the accuracy requirement is 1 m, then the sampling rate will need to be less than ~80 meters over flat areas and ~40 meters for a modestly undulating surface. In this example, in contrast, this goal cannot be reached for mountainous regions. While a DEM derived from satellite data was used in these experiments, this method is generic and allows for support of the optimization of the sampling rate of aerial and/or other data acquisition systems to any desired level of accuracy.

CONCLUSIONS AND FUTURE WORK

In this paper, we present a procedure to estimate surface errors introduced by using different sampling rates. The analysis is based on spatial frequency domain processing, and it estimates the RMSE as a function of the sampling distance. Note that the condition to meet the Nyquist criterion (in other words, the spatial surface signal is band limited) is assumed in this study. Three types of DEMs obtained from SRTM mission data and representing different major terrain categories were used for validating the method. The research results indicate that for a given accuracy requirement, the sampling distances are correlated to the surface category. In addition, the relationship between errors introduced by downsampling the surface and the sampling rate is established. This can provide input to data acquisition planning and, even, to DEM archival considerations. Future work will address a terrain types to provide statistically relevant classifications for general data acquisition planning.

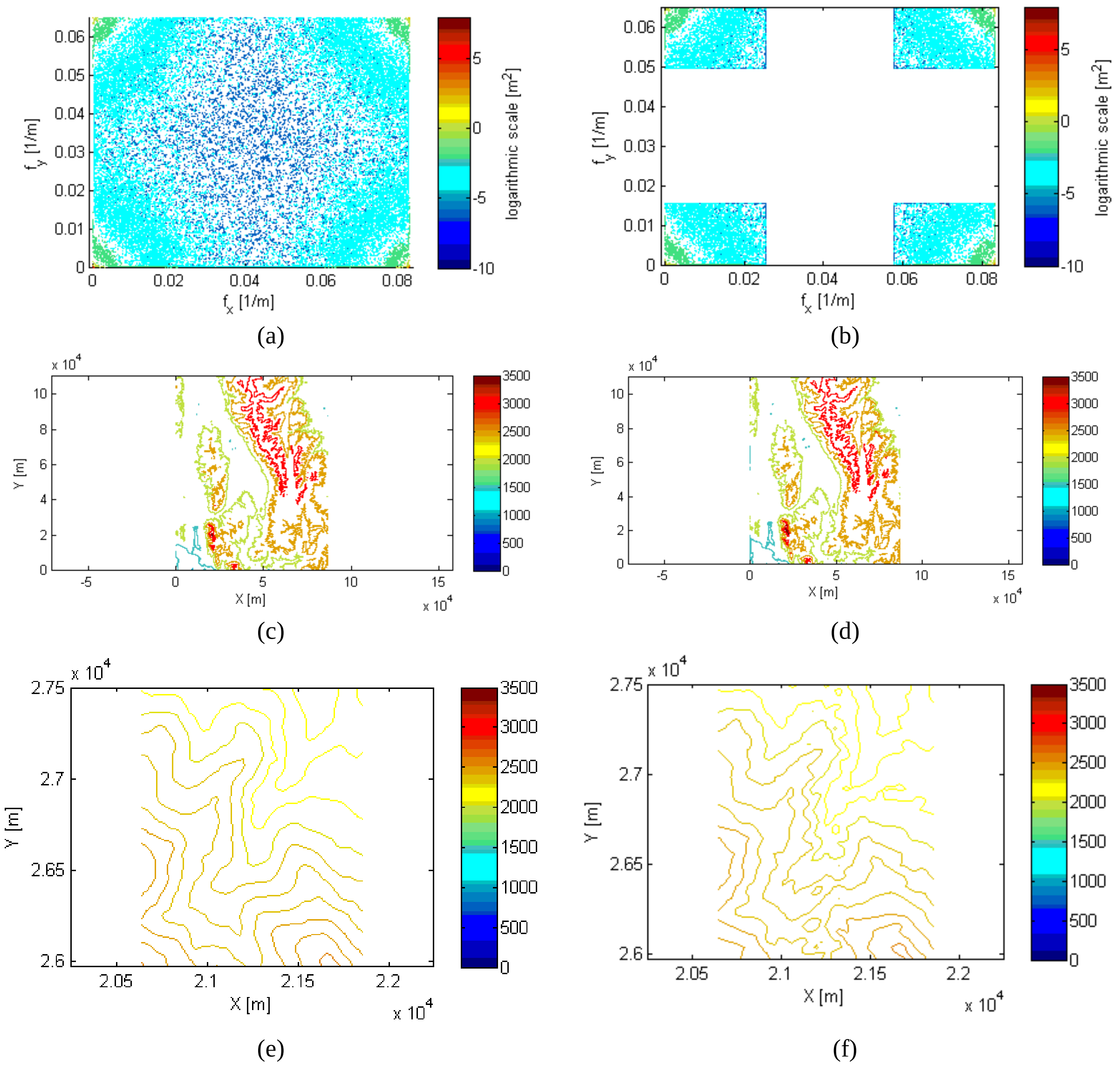


Figure 2. The processing steps illustrated for DEM-3: (a) spatial frequency representation, (b) truncating the spatial spectrum, (c) original DEM, (d) reconstructed DEM, (e) enlarged area of original DEM, and (f) enlarged area of reconstructed DEM.

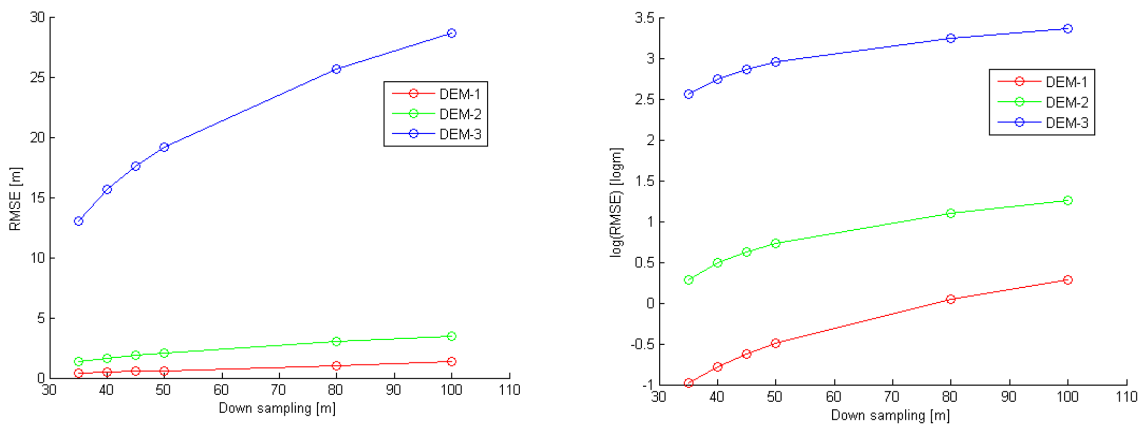


Figure 3. Reconstruction errors (linear and logarithmic) of the DEMs at different downsampling rates.

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