ANALYSIS OF EPIPOLAR GEOMETRY MAPPING RELATION FOR DIGITAL FRAME CAMERA IMAGES

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ABSTRACT

Digital aerial frame cameras are widely used in mapping applications. As one most effective and frequently used constraint in digital photogrammetry research and practices, however, epipolar geometry relation of digital frame camera is not investigated deeply. In this paper, analytical transforming relation between center projective image (CPI) obtained by digital frame camera and epipolar image (EPI) is derived, based on which some fruitful result are revealed. The contributions of this paper are as follows. First the monotone mapping relation from CPI to EPI is obtained, which lays the foundation of two calibration algorithms in epipolar image generation. One is the fast indirect rectification per block algorithm; the other is direct rectification algorithm for any sub-region in CPI. Then a critical epipolar line is proved to be existed in CPI. The coordinates of epipolar point that all epipolar lines intersect at in CPI, are deduced thirdly. From that epipolar point, it is very easy to get slant epipolar line that goes through any image point in CPI.

KEYWORDS: digital frame camera, center projective image, epipolar image generation.

INTRODUCTION

As pointed out by Heipke, image matching has long been and is still one of the most challenging techniques in digital photogrammetry research and applications[1], it is a ill-posed problem. To make it into a well-posed one, it is necessary to introduce other conditions and restrictions. Epipolar geometry is the most effective and frequently used constraint in image matching.

In stereo image pairs, epipolar lines are a powerful constraint for conjugate entity matching. Fig.1.1 illustrates the concept. The epipolar plane is defined by the two projection center C', C'', and the object point P. The epipolar lines e', e'' are the intersections of the epipolar plane with the image plane. The epipolar plane contains conjugate points which must lie on the corresponding epipolar line. Epipolar geometrical condation reduce the search space dramatically. If matching entity in one image is chosen then the epipolar line in the other image can be computed, provided the stereopair is oriented.

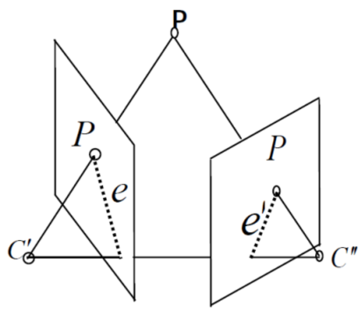
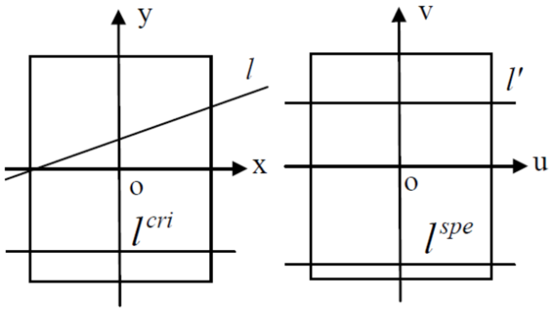


Figure 1.1 Epipolar geometry of a stereopair

Usually, epipolar lines are not parallel to the x-axis of phto-coordinate system of digital frame camera images. Quite often it is desirable to transform the digital frame camera images called center projective image (CPI) in such a way that the epipolar lines become parallel to the rows of the digital images. This process is called epipolar image(EPI) generation or calibration(Figure 1.2).



(a) CPI phto-coordinate (b) EPI phto-coordinate

Figure 1.2 Mapping relation between CPI and EPI

In calibrating epipolar images, what do the analytical models between CPI and EPI look like? What kind of mapping rule from CPI to EPI to obey? How could one get the fast algorithm of epipolar image calibration, which we will call block-to-block calibration method in this paper? Beside the traditional indirect generation algorithm, is there any direct calibration method? Commonly, epipolar image is rectified for hole frame CPI, which is not necessary and very time consuming in matching sub-region of stereo image pairs. Can one generate regional epipolar image of any polygon area in CPI? These problems have not been investigated deeply and solved well.

In this paper, analytical transforming relation between CPI obtained by digital frame camera and EPI is derived, based on which some fruitful result are revealed. The contributions of this paper are as follows. First the monotone mapping relation from CPI to EPI is obtained, which lays the foundation of two calibration algorithms in epipolar image generation. One is the fast indirect rectification per block algorithm; the other is direct rectification algorithm for any sub-region in CPI. Then a critical epipolar line is proved to be existed in CPI. The coordinates of epipolar point that all epipolar lines intersect at in CPI, are deduced thirdly. From that epipolar point, it is very easy to get slant epipolar line that goes through any image point in CPI.

This paper begins with the derivation of monotone mapping relation from CPI to EPI in subsection 1. Then in subsection 2, a critical epipolar line is proved to be existed in CPI. The critical epipolar line is referred to the one that is unusually parallel to the x-axis of phto-coordinate system of digital frame camera images. Moreover, the coordinates of epipolar point that all epipolar lines intersect at in CPI, are computed. Based on above result, the fast block-to-block direct calibration algorithm and the regional indirect calibration algorithm are presented. Finally, the experimental results and conclusions are provided.

ANALYSIS OF MAPPING RELATION BETWEEN CPI AND EPI

From figure 1.2, for a image point (x,y) in CPI, supposing its correspond point coordinate is (u,v) in EPI , then we can get equation(1) after relative orientation,

$$\begin{cases} u = u(x,y) = (-f)\cdot\dfrac{a_1x+a_2y-a_3f}{c_1x+c_2y-c_3f} \\ v = v(x,y) = (-f)\cdot\dfrac{b_1x+b_2y-b_3f}{c_1x+c_2y-c_3f} \end{cases}, \quad \begin{cases} x = x(u,v) = (-f)\cdot\dfrac{a_1u+b_1v-c_1f}{a_3u+b_3v-c_3f} \\ y = y(u,v) = (-f)\cdot\dfrac{a_2u+b_2v-c_2f}{a_3u+b_3v-c_3f} \end{cases} \tag{1}$$

where f is focal length of digital frame camera, a_i, b_i are the elements of rotation matrix R. For a horizontal line l': $v = v_0$, usually called a epipolar line in EPI (here horizontal means that this line is parallel u-axis of phto-coordinate system of EPI), there is always a slant line l corresponding with it in CPI (here slant means that l is not parallel x-axis of phto-coordinate system of CPI), and vice versa.

$$l: \qquad y = kx + b \qquad (2)$$

where, $k = -\frac{b_1 f + c_1 v_0}{b_2 f + c_2 v_0}$, $b = f \cdot \frac{b_3 f + c_3 v_0}{b_2 f + c_2 v_0}$. The following proposition shows that k is a monotone function of v.

Proposition 1 Given a CPI of digital frame camera, for a horizontal epipolar line l': $v = v$ in EPI, its corresponding line l as equation 2, then k is a monotone function of v.

Proof. From equation 2, we have

$$k'_v = -\frac{c_1(b_2 f + c_2 v) - (b_1 f + c_1 v)c_2}{(b_2 f + c_2 v)^2} = \frac{(b_1 c_2 - b_2 c_1) f}{(b_2 f + c_2 v)^2} \qquad (3)$$

As elements of rotating matrix R meet following equations:

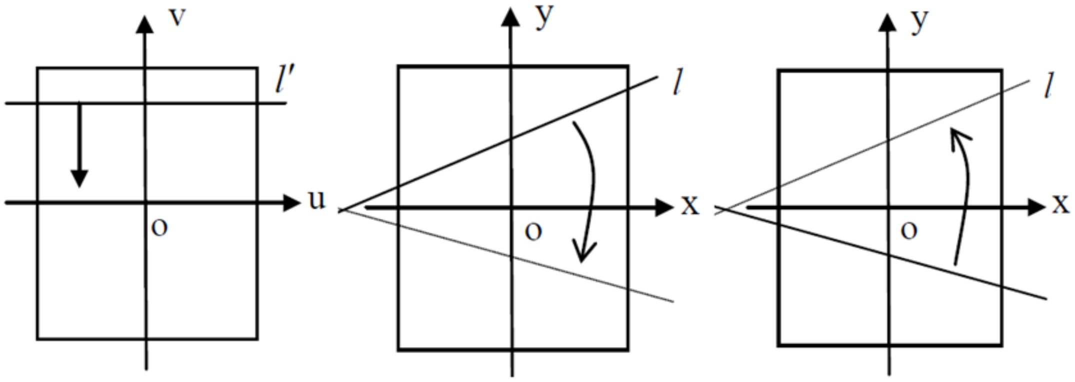
$$a_3 = \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix} = b_1 c_2 - b_2 c_1 \qquad (4)$$

Then we have:

$$k'_v = \frac{a_3 f}{(b_2 f + c_2 v)^2} \qquad (5)$$

It is easy to see that the sign of k'_v is same as the sign of a_3. After relative orientation, the directional cosine a_3 is definite. Accordingly, if a_3 is positive, then k is a monotone increasing function of v; if a_3 is negative, then k is a monotone decreasing function of v.

From proposition 1 it can be seen that if $a_3 > 0$, then when the epipolar line in EPI l' moves down scanning the hole image plane of EPI, the slant epipolar line l in CPI rotates down when $a_3 > 0$ around epipolar point, and scans up when $a_3 < 0$ around epipolar point. This monotone transform lay the foundation of block-to-block and parallel algorithm algorithm of epipolar image generation (see subsection 3).



(a)EPI image plane, epipolar line moving down

(b)CPI image plane, epipolar line moving down when $a_3 \geq 0$

(c)CPI image plane, epipolar line moving up when $a_3 < 0$

Figure 2.1 Monotone mapping relation between CPI and EPI image plane

Usually, epipolar lines are not parallel to the x-axis of phto-coordinate system of digital frame camera images. From proposition 2, one can find a special epipolar line called critical line that is parallel to the x-axis of phto-coordinate system of CPI image plane. This is the proposition 2.

Proposition 2 There exists a critical epiolar line in CPI image plane.

Proof. In proposition 1, let gradient k equals to zero, there is special line l^{spe} in EPI:

$$l^{spe} \ : \ v = -\frac{b_1}{c_1} f \qquad (6)$$

Its corresponding epipolar line l^{cri} in CPI is:

$$l^{cri} \ : \quad y = b = f \cdot \frac{b_3 f + c_3 v^{spe}}{b_2 f + c_2 v^{spe}} = -\frac{a_2}{a_3} f \qquad (7)$$

Obviously, l^{cri} is parallel to x-axis of CPI image plane, see figure 1.2.

Proposition 3 The coordinates $\left(x_{epi}, y_{epi}\right)$ of epipolar point E that all epipolar lines intersect at in CPI are $\left(-\frac{a_1}{a_3} f, -\frac{a_2}{a_3} f\right)$.

Proof. From equation 7, it is known that $y_{epi} = -\frac{a_2}{a_3} f$. For epipolar line $v = 0$ in EPI, the relevant epipolar line in CPI is:

$$b_1 x + b_2 y - b_3 f = 0 \qquad (8)$$

et $y = y_{epi} = -\frac{a_2}{a_3} f$ in (8), thus infer that

$$x_{epi} = -\frac{a_1}{a_3} f \qquad (9)$$

FAST ALGORITHMS OF EPIPOLAR IMAGE GENERATION

Usually, there are two kinds of epipolar image calibrating algorithm, namely direct calibration algorithm and indirect one. The most commonly used is indirect calibration algorithm. Due to the large volume of digital frame camera images, the calibration of whole frame epipolar image not only takes up a lot of memory, and very time-consuming. On the basis of proposition 1, a fast calibration algorithm can designed, it generates epipolar image in block-to-block and takes a small amount of memory.

Fast indirect calibration algorithm per block

On the bases of proposition 1, the mapping transform between CPI and EPI is monotone, as shown in figure 2.1. Thus for a rectangular area A that located between two epipolar lines $v = v_1, v = v_2$, $v_2 < v_1$ in EPI(Figure 3.1,(b)), the relevant region B that is necessary in calibrating calculation in CPI, has to be contained between two slant lines l_1, l_2 (Figure 3.1,(a)). Accordingly, the calibration of whole epipolar image can be done per block and parallel. The resultant per block algorithm is:

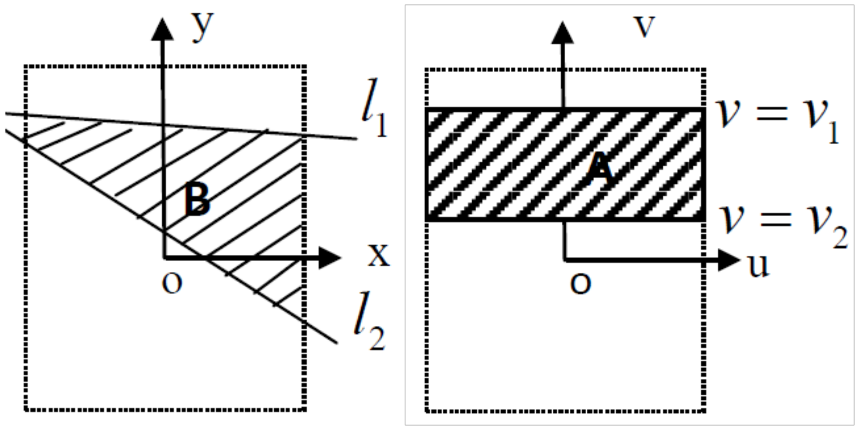
ALGORITHM 1:

Step 1: Taking one rectangular area A in EPI.

Step 2: Calculating correspondent area B in CPI, reading B into memory.

Step 3: For every pixel (u,v) in A, calculating coordinates (x,y) according to equation 1 .

Step 4: Interpolating gray scale value g of pixel (x,y), and then assigning g to (u,v).



(a) correspondent area in CPI (b) a rectangular area in EPI

Figure 3.1 Monotone mapping relation between CPI and EPI

Direct calibration algorithm for any sub- region in CPI

In order to calibrate directly any sub-region in CPI, it has to determine the next adjacent epipolar line after finished the previous line calibration. Only when the angle between any two successive slant epipolar lines in CPI is known, is one able to calibrate successively every line in EPI, and this process is done directly in CPI.

Given any two adjacent epipolar lines in EPI, $v = v_1$ and $v = v_1 + \Delta$, Δ is sampling distance as same as CPI. The gradients of correspondent these two slant lines in CPI are:

$$k_1 = -\frac{b_1 f + c_1 v_1}{b_2 f + c_2 v_1} \qquad k_2 = -\frac{b_1 f + c_1 v_2}{b_2 f + c_2 v_2} \qquad (10)$$

$$k_2 - k_1 = \frac{a_3 f \Delta}{(b_2 f + c_2 v_1 + \Delta) \cdot (b_2 f + c_2 v_1)} \triangleq \beta(v_1) \qquad (11)$$

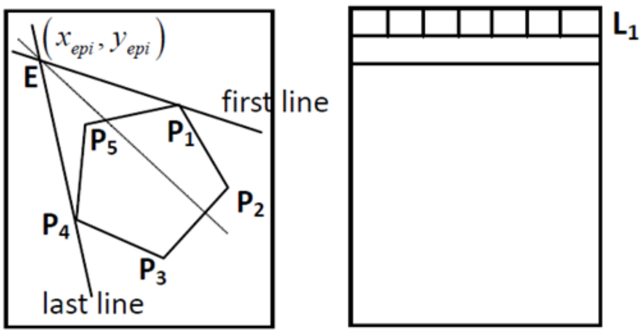
Note first equation in (10), it follows:

$$v_1 = -\frac{b_1 + k_1 b_2}{c_1 + k_1 c_2} f \qquad (12)$$

Take (12) into (11), it has:

$$k_2 = k_1 + \beta(v_1) = k_1 + \frac{\Delta \cdot (c_1 + k_1 c_2)^2}{a_3 f - c_2 \Delta (c_1 + k_1 c_2)} \triangleq k_1 + \lambda(k_1) \qquad (13)$$

Equation (13) shows that the gradient increment of next adjacent epipolar line in CPI is a function of gradient of previous epipolar line. Once the previous epipolar line is determined, the adjacent epipolar line is easy to find by k_2 and epipolar point E $\left(x_{epi}, y_{epi}\right)$. The direct calibration algorithm for any sub-region in CPI is consequently as follows (see figure 3.2). The algorithm 2 is summarized based above analysis.



(a) a polygon area in CPI (b). EPI of the polygon in (a)

Figure 3.2 Schematic diagram of sub-region calibration in CPI

ALGORITHM 2:

Step 1: Finding out the most border points P_1~P_5 of a polygon area in EPI, and calculating the width W along x-axis of the polygon.

Step 2: For i=1, determining the first line l_i that goes through point E and P_1, computing the gradient k_i of that line.

Step 3: Resampling W/Δ pixels along l_i, and putting them in line array L_i in EPI.

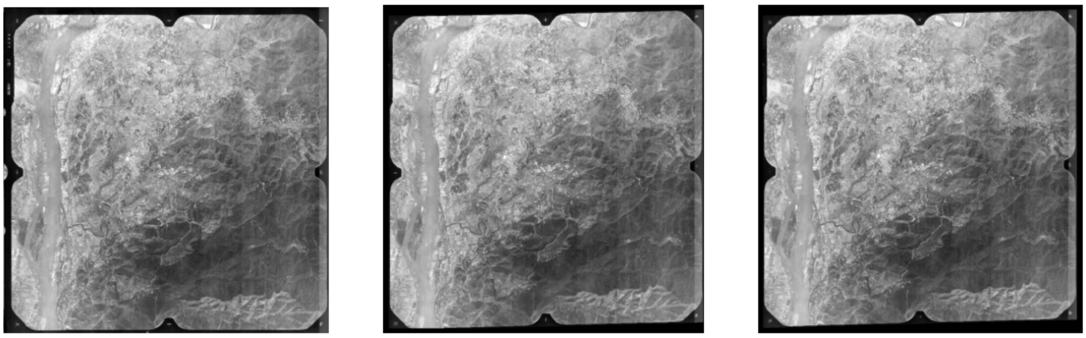
Step 4: Calculating next gradient k_{i+1} of next adjacent epipolar line by (13), and then getting l_{i+1}.

Step 5: Turning to Step 3.

Step 6: Until the last line.

EXPERIMENTAL RESULTS AND CONCLUSIONS

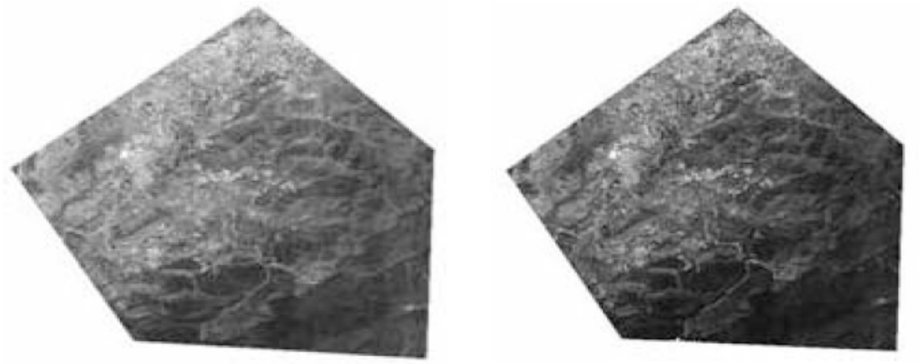
A digital aerial image scanned from hardcopy image obtained by frame camera was used in experiments, several experimental results are obtained by using above two calibration algorithm In figure 4.1, one of these results is showed in figure 4.1. In figure 4.1, (a) is original digital frame CPI , (b) is EPI calibrated by proposed per-block indirect algorithm, it is seamless and exact same as one rectified by traditional indirect algorithm. More important, the time that proposed per-block algorithm used is as one and half or less as that traditional one used, see table.



(a)Original CPI (b)EPI calibrated by per-block indirect algorithm (c)EPI calibrated by traditional indirect algorithm

Figure 4.1 Comparison of proposed algorithm and traditional one

For epipolar image rectification of any sub-region in CPI, traditional indirect method is not able to do anything. Because our algorithm 2 above is capable of calculating gradient increment of epipolar line in CPI, it can calibrate directly any sub-region area in CPI to regional epipolar image area , figure 4.2 presents the experimental result.



(a)a polygon region in CPI (b)EPI of the polygon region in CPI

Figure 4.2 EPI of a polygon region in CPI

From above analysis and experimental results, it can be seen that the methods in this article has better performance than traditional methods, and the algorithm 2 can generate EPI for any polygon region in CPI, fill the gaps left by traditional algorithm. These algorithms can be used in digital photogrammetry research and practices.

REFERENCES

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