

# Nested Regression Based Optimal Selection (NRBOS) of Rational Polynomial Coefficients

Long Tengfei, Jiao Weili, and He Guojin

## Abstract

Although the rational function model (RFM) is widely applied in photogrammetry, the application of terrain-dependent RFM is limited because of the requirement for numerous ground control points (GCPs) and the strong correlation between the coefficients. A new method, NRBOS, based on nested regression was proposed to select the optimal RPCs automatically and to gain stable solutions of terrain-dependent RFM using a small amount of GCPs. Different types of images, including QuickBird, SPOT5, Landsat-5, and ALOS, were involved in the tests. NRBOS method performed better than conventional methods in estimating RPCs, and even provided a reliable solution when less than 39 GCPs were used. Additionally, the test results showed that the simplified RPCs are almost as accurate as the vendor-provided RPCs. Consequently, in favorable situations such as when the orientation parameters of the satellite are not available or are not sufficiently accurate, the proposed method has the potential to take the place of the regular terrain-independent RFM.

## Introduction

The rational function model (RFM) with 78 rational polynomial coefficients (RPCs) is completely a mathematical model, which approximately describes the imaging process in photogrammetry and remote sensing. Terrain-independent RFM constitutes a comprehensive reparameterization of the rigorous sensor model, and is widely applied in high resolution image products. Terrain-dependent RFM, on the other hand, is hardly used because of the requirement for numerous observation data (ground control points (GCPs)), and the strong correlation between the coefficients. Actually, terrain-dependent RFM provides a useful approach to rectify remotely sensed images without knowing the position and orientation information of specific sensor. However, as 78 RPCs of the RFM are strongly correlated, stable, and precise solutions of the RPCs are difficult or even impossible to achieve (Lin and Yuan, 2008). The objectives of this research are to find a robust approach to estimate RPCs and to make use of the terrain-dependent RFM.

Many studies have been carried out on the topic of RFM during the recent decades. OGC (1999) has normalized the range of the image and object space coordinates of RFM to  $-1$  to  $+1$ , and effectively enhanced the condition number of the normal equation matrix. Tao and Hu (2000) have studied the RFM comprehensively and have proposed to strengthen the solution of RFM using Tikhonov regularization and the L-curve method. Yuan and Lin (2008) have compared the results of several methods for solving RPCs including ridge trace method, L-curve method, empirical formula method, and generalized ridge estimate method; they have verified the validity of L-curve method. In addition, Levenberg-Marquardt method

(Tao and Hu, 2001) and singular value decomposition method have been applied to solve RPCs (Fraser *et al.*, 2006). Among all these methods, the ridge estimation method (especially the L-curve method) is the most widely used approach. However, there are still some problems in solving RPCs using the existing methods. For example, ridge estimate is a biased estimate, which requires numerous GCPs to solve RPCs.

Solving RPCs is a problem of multiple regression analysis. The problem of multicollinearity results in an ill-posed normal equation, and the ordinary least squares (OLS) estimation does badly in achieving a stable and reliable solution. In order to solve the problem of multicollinearity, “variable selection (Draper *et al.*, 1966)” and “ridge estimation (Hoerl and Kennard, 1970)” are usually used to improve the OLS. Variable selection can simplify the original model by selecting a subset of variables from the original set of variables which give the most significant response to the regression, and therefore the multicollinearity of the model can be reduced after the variable selection (Guyon and Elisseeff, 2003). Both of the two methods have drawbacks. Variable selection provides interpretable model but can be extremely variable because it is a discrete process. Ridge regression is a continuous process that shrinks coefficients and hence is more stable. However, it does not set any coefficients to 0 and hence does not give an easily interpretable model (Tibshirani, 1996). In addition to the methods mentioned above, in statistics, some improvements of ridge estimation method (Bashtian *et al.*, 2011; Jurczyk, 2012; Kibria and Saleh, 2012; Park and Yoon, 2011) have been proposed in recent years. However, they still need a large number of observation data.

Variable selection is a non-deterministic polynomial (NP) problem whose search space is very large (Amaldi and Kann, 1998). During the past decades, hundreds of methods have been proposed to solve this problem, including genetic prediction, decision tree prediction, Bayes prediction, least square prediction, and support vector machine prediction (Guyon and Elisseeff, 2003), and all these methods need a huge computation. Greedy search strategy can greatly reduce the search space and thus improve the efficiency of the algorithm. The greedy search methods are mainly divided into three categories: forward selection, backward elimination, and stepwise regression analysis. Nonetheless, there are common disadvantages in these methods. Not all the possible combinations of variables are taken into account, and the results which greatly depend on the evaluation criterion are usually not the optimal solution. In addition, the variable transformation, such as principal component analysis, partial least

Photogrammetric Engineering & Remote Sensing  
Vol. 80, No. 3, March 2014, pp. 261–269.  
0099-1112/14/8003–261

© 2014 American Society for Photogrammetry  
and Remote Sensing  
doi: 10.14358/PERS.80.3.261

The Institute of Remote Sensing and Digital Earth (RAD),  
Chinese Academy of Sciences, No.9 Dengzhuang South Road,  
Haidian District, Beijing 100094, China (wljiao@ceode.ac.cn).